HOUGHTON MIFFLIN

ALGEBRA ACTION

Courseware for Algebra 1

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Courseware for Algebra 1

Teacher's Manual

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I. INTRODUCTION

ALGEBRA ACTION: Courseware for Algebra 1 is a 4-disk software program designed to supplement a first-year algebra textbook. The courseware is correlated with the algebra curriculum, but is not intended to replace the textbook. Through tutorials, review, practice, and games, the program highlights the main topics of algebra.

Each disk contains two types of presentations:

FOCUS—For each topic, tutorials explain and illustrate the concept, repeating or advancing in accordance with student direction. This mode also includes some guided interaction with the student.

ACTIVITIES—The student may practice applying the FOCUS concepts by trying randomly generated exercises in the Practice Problems for each topic. Similar problems are also incorporated into games and other activities that allow the student to improve both speed and accuracy.

Some topics also include a REVIEW section, to prepare students for the tutorial to follow. ALGEBRA ACTION is menu-driven, so that students can easily select a topic for study, shift back and forth between FOCUS and ACTIVITIES sections, and change topics at any time.

The four disks that accompany Algebra 1 cover the following topics:

DISK 1—Variables;
Solving Linear Equations

DISK 2—Multiplying and Factoring Polynomials;
Fractions, Ratios, and Proportions

DISK 3—Graphing Linear Equations;
Introduction to Functions

DISK 4—Inequalities;
Irrationals

A detailed topic outline begins on page 4.

II. GETTING STARTED

EQUIPMENT NEEDED

- Apple® II, II+, IIc or IIe computer, with at least 64K memory
- one disk drive
- monitor (Although a color monitor is not essential, ALGEBRA ACTION is enhanced by a color screen; sound use is minimal and is not required.)

CARE OF DISKS

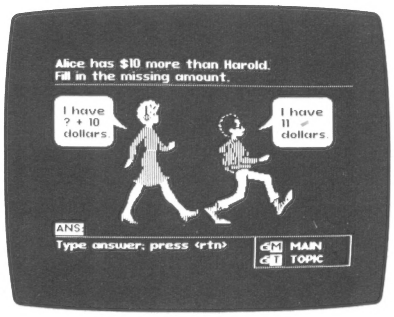
ALGEBRA ACTION is copy-protected, but back-up copies of all the disks are provided. Save the extra copies in a safe place, using only one copy. Store all disks away from heat, dust, and magnetic devices. Handle the disks by the protective sleeve only, without touching the exposed parts of the disk. Do not insert or remove a disk from your disk drive while the drive's red light is on; doing so could permanently damage the disk.

LOADING ALGEBRA ACTION

1. With the label side up, insert Disk 1 into the disk drive. If you have a two-drive system, use drive 1.
2. Turn on the computer and the monitor.
   The red light on your disk drive will go on, and you will hear whirring noises while the program is loading. In a few seconds, the opening screens will appear, followed by the main menu.

WALKING THROUGH THE PROGRAM

The main menu screen shows a bulletin board with topics available on this disk; also, the last selection on this menu is a list of topics contained on the other three disks. A checkmark is next to Variables. Use the ↑ ↓ keys (Apple IIc, IIe) or the A Z keys (Apple II, II+) to move the checkmark up and down. Move the checkmark back to Variables and press <return> to select this main topic. Now the topic menu for Variables appears, with a checkmark next to the FOCUS section, What Are Variables?; press <return> again to choose this item.



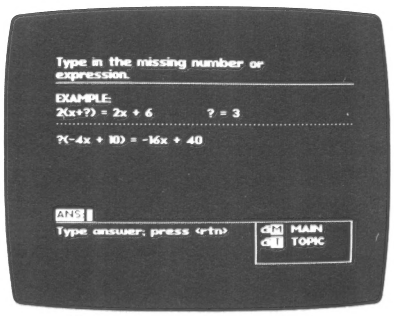
As in all tutorials, you proceed through this lesson by pressing →, then <return>, to move to the next screen; to review a previous screen, press ←, then <return>. Advance through the next few screens until you are asked to fill in the missing amount. The prompt tells you to type your answer, then press <return>. Note the flashing cursor, an additional cue to indicate that your input is expected. Type 11, then <return>.

When asked for an answer on the next screen, give an incorrect response. The computer responds with an INCORRECT message and writes the correct answer. Continue through the next few screens, until you are asked to enter 5 integers. The prompt sets limits for your integers and asks you to input one integer and press <return>. The prompt remains until all 5 integers have been entered. The program then uses your integers to generate even numbers and, on the next screen, odd numbers.

Next, you are asked to tell whether the numbers generated by $2n + 2$ are even or odd. For nonnumerical answers like this, only the first letter of the response is necessary. The program accepts either a capital or a lowercase letter. You may also request HELP with this question, as the prompt indicates. To get HELP, press ◌ and ? at the same time (<escape>? on the Apple II, II+).

Try the next few screens, until the first A Challenging Thought appears. A Challenging Thought screens, included in some of the tutorials, are intended to stimulate further thinking, but no response is to be entered into the computer. (Answers are given in the Detailed Topic Outline.) Now use the ◌T command (<escape>T on Apple II, II+) to return to the Variables menu.

Then select Distributive Axiom and Combining Like Terms by using the ↑ ↓ keys (A Z on Apple II, II+). The first screen of this section asks for an area. Although ALGEBRA ACTION allows some variability in the form of the responses it accepts, it is best to give answers in the simplest form—in this case, $2n$. Note that it is not necessary to include a multiplication symbol with variables; multiplication is implicit. Proceed through several screens, then watch the development of the screen labeled, "Multiplication is distributed over every term in the parentheses." Here the program emphasizes the use of the distributive axiom by underlining each term as it is being multiplied. On the next screen (see left), you are asked to complete the given examples. After working the problems, you may ask for another example, as the prompt indicates. Press A and then <return> to see a new example. Because the examples are randomly generated, you may repeat as often as you like. This type of practice is embedded within all the tutorial sections, so that students may gain confidence before proceeding to new material.



When you reach the end of the tutorial, the program will return to the topic menu. You may repeat the same section, try another tutorial, or select one of the ACTIVITIES. You may also exit to the main menu from any point in ALGEBRA ACTION by pressing ◌M (<escape>M on Apple II, II+). To change to another disk, insert the new disk in the disk drive and press ◌<control><reset> (<reset> on Apple II, II+).

III. USER INPUTS

PROGRAM CONTROL INPUTS

You control the flow of the program in several ways. Prompts at the bottom of the screen indicate the options available to you at that point. Except for the Exit controls, which are always in effect, only those commands whose prompts are showing may be used. The table below summarizes the program control commands.

NOTE: A comma between key symbols (←, <return>) in the table indicates that keys are to be pressed successively; if no comma is shown (<shift>M), they are to be pressed simultaneously.

Apple II, II+ command	Apple IIc, IIe command	Function
M	<shift>M	Express key. If pressed before end of copyright message, program skips introductory screens and goes directly to main menu.
←, <return>	←, <return>	Return to previous screen. May cause skipping back 2 or 3 screens in order to start at the beginning of a thought or sentence.
→, <return>	→, <return>	Advance to next screen.
A, <return>	A, <return>	Give another example.
<escape>M	Ó M	Exit to main menu for disk.
<escape>T	Ó T	Exit to menu for current topic.
<escape>?	Ó ?	Show a HELP screen.
<reset>	Ó <control><reset>	Restart. Can be used to change disks or restart current disk.

RESPONSE INPUTS

Throughout all sections of Algebra Action, you are invited to interact with the program. When the answer is a specific word, you need only enter the first letter—which is highlighted—followed by <return>. You may enter the entire word, but if it is misspelled, it may be treated as an incorrect response. Either capital or lowercase letters may be used.

For numerical or algebraic responses, some variation in input is accepted. When a specific form is illustrated, however, it is important to follow that form. In general, the simplest possible form is usually the safest one to use. All user input may be corrected before <return> is pressed. Use ← to backspace, retype as necessary, then press <return>.

Special attention should be given to exponents. To enter $3x^2$, for example, type $3x$, then press ^ (<shift>6 on Apple IIc, IIe; <shift>N on Apple II, II+). The cursor moves up to the exponent position; after you type the number 2, the cursor immediately returns to the base line.

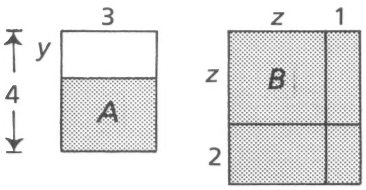
In general, the program rejects inappropriate input, either by ignoring such keys and erasing them when <return> is pressed or by treating the answer as incorrect. Errors may result from attempting to perform operations with numbers exceeding 32,768 (2^{15}). Further details and examples of inputs may be found in the Detailed Topic Outline.

IV. DETAILED TOPIC OUTLINE

DISK 1, PART 1: Variables

FOCUS
- What Are Variables?
- Distributive Axiom and Combining Like Terms
- Word Problem Warm-ups

ACTIVITIES
- Drive to Glory
- Practice Problems



WHAT ARE VARIABLES?

OBJECTIVE: To use variables to describe patterns in numbers.

INTRODUCTION: It is sometimes difficult for students to grasp the concept of a variable expression representing a general relationship. Here diagrams help the student make the connection between the variable expression and the quantity it represents.

You may wish to review odd and even integers, asking if students know any rules for generating an even number from any given integer. For example, multiplying by 10 always produces an even number. Ask if students can think of a rule to generate an odd number from any given integer. (Multiplying by an even number, then adding an odd number is one way.)

SUGGESTED EXTENSION: Have students try writing variable expressions for diagrams involving two or more operations. For example, in the diagrams at the left, area A is given by $3(4 - y)$ and area B is given by $(z + 1)(z + 2)$.

CHALLENGING THOUGHTS: (1) When n is even, $3n + 5$ is odd; when n is odd, $3n + 5$ is even.
(2) The area is $2ax$.

DISTRIBUTIVE AXIOM AND COMBINING LIKE TERMS

OBJECTIVE: To use the distributive axiom to simplify expressions.

INTRODUCTION: Practice in this section not only asks the student to complete distributions such as $2(5x + 7) = ?$, but also to complete statements such as $9(-x + ?) = -9x + 54$ and to simplify expressions such as $4z - 52z$.

Students should understand the use of parentheses in determining the order of operations. For example, they should recognize that $3x + 5$ is not the same as $3(x + 5)$. You may choose to review this idea before using the program.

PROGRAM NOTES: Not all possible algebraic forms of answers will be accepted as correct. For $2n$, for example, $n + n$ will be accepted, but $20n/10$ will not be. Note that in questions of the form $8z + 13z = ?z$, only the numerical coefficient is to be input; 21 is correct, but $21z$ will be treated as an incorrect response.

WORD PROBLEM WARM-UPS

OBJECTIVE: To interpret relationships expressed in words and to translate them into numerical expressions.

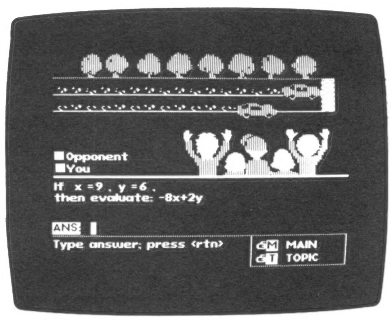
INTRODUCTION: Students are given a series of facts and asked to answer related questions. This section lays a foundation for the more complex task of writing equations.

PROGRAM NOTES: Responses in this section require only the first letter of the answer, followed by <return>. You may enter the entire word, using either capital or lowercase letters, but a misspelled answer may be treated as an incorrect response.

SUGGESTED EXTENSION: After completing this tutorial, students may enjoy trying to interpret other types of word problems. See, for example, pages 56-58 and pages 62-63 in Book 1 of Algebra, Structure and Method. You may also wish to have students begin organizing information in chart form, as on pages 119 and 123.

ACTIVITIES

OBJECTIVE: To improve speed and accuracy in evaluating expressions.



INTRODUCTION: Both the Drive to Glory game and the Practice Problems section offer the student the opportunity to practice evaluating expressions at five possible levels of difficulty.

LEVEL	SAMPLE PROBLEM Evaluate:	
1	$4x - 1$	(if $x = 8$)
2	$-x + 3y$	(if $x = 1, y = 10$)
3	$4xy - 5x$	(if $x = 9, y = 2$)
4	$3y + z - 3x$	(if $x = 10, y = 4, z = 8$)
5	$xyz - xy$	(if $x = 3, y = 6, z = 6$)

In Drive to Glory, you race a car against an opponent, who drives at the fixed speed you pick (between 30 and 150 mph). You move your car (bottom one on screen) by solving problems. When you give a correct response, a checkmark will appear, and your car will move forward. It takes 5 correct answers to move to the finish line. If you reach the finish line before your opponent, the fans cheer and you are congratulated. Win or lose, you are given a chance to play again, as many times as you wish.

The Practice Problems section has the same types of exercises, at the corresponding levels of difficulty, as in Drive to Glory. Here there are no time constraints, so that students may practice simply to improve their skill and understanding, without the pressure of the race. When the student gives an incorrect answer in this section, the INCORRECT flag appears, and the program gives the correct answer.

DISK 1, PART 2:
Solving Linear Equations

REVIEW
- Signed Number Practice

FOCUS
- Using One Operation
- Using Several Operations

ACTIVITIES
- Drive to Glory
- Practice Problems

SIGNED NUMBER PRACTICE

OBJECTIVE: To review the four basic operations with signed numbers.

INTRODUCTION: The student practices adding, subtracting, multiplying, and dividing signed numbers by solving exercises such as:

$$\frac{10}{-2} \cdot \frac{8}{8} = ?$$

PROGRAM NOTES: Answers should be given in simplest possible form: a sign (if necessary), followed by digits, with no more than 5 characters altogether.

USING ONE OPERATION

OBJECTIVE: To apply an inverse operation to solve a linear equation.

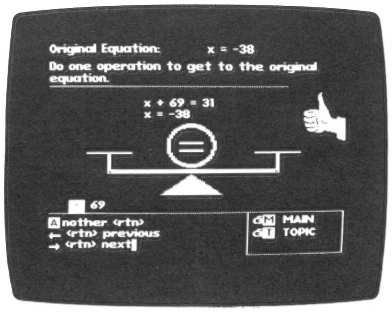
INTRODUCTION: In this tutorial, a balance is used to demonstrate the effect of performing an operation on an equation. The student learns to solve an equation by applying an inverse operation. Note that the computer inserts a space between a variable and its fractional coefficient. Thus the equation $\frac{4}{5}x = 7$ is displayed as 4/5 $x = 7$.

PROGRAM NOTES: In the practice section, you are asked to derive the original equation by performing one (inverse) operation. First, select an operation by pressing the ←, → keys until the cursor is to the left of the desired operation; then press <return>. The program will prompt you for the value to be used with the operation. This number may have a sign, and the slash (/) may be used for fractions. (Mixed numerals or decimal forms will not be accepted.) You may choose to add −13/4 or subtract 13/4, for example; or you may divide by 3 or multiply by 1/3.

As usual, input numbers may not exceed 32,768. If an operation results in a calculation that exceeds this limit, an Out of Range message is printed, accompanied by a "beep," and the original equation is repeated. The original equation will likewise be displayed, along with an appropriate message, if you attempt to multiply or divide by zero.

The thumbs-up sign indicates that the original equation was derived; if your operation yields another form of the equation, a message to that effect is displayed.

These restrictions also apply in the equation-solving section (where no balance is showing). You are again asked to select an operation and enter a value to be used, as above. But



here you may take as many steps as you like to solve the equation—although these equations can also be solved in just one step. To see the solution of any equation, request a HELP screen (? on Apple IIc, IIe; <escape>? on Apple II, II+).

USING SEVERAL OPERATIONS

OBJECTIVE: To solve linear equations involving more than one operation.

INTRODUCTION: The practice section includes problems at five levels of difficulty.

LEVEL	SAMPLE PROBLEM Solve:
1	$15x = -1/10$
2	$1/2\ x + 49 = 22$
3	$10 \cdot (x - 26) + 8 \cdot (x + 19) = 126$
4	$-10 \cdot (6x - 9) = 3 \cdot (-4x - 20)$
5	$(10/3) \cdot (x + 9) = (2/5) \cdot (-10x + 24)$

This section uses a unique routine called the Solver, which allows the student to choose from a number of possible steps to solve the equation. Besides applying the four basic operations, the Solver can combine like terms, apply the distributive axiom, or return to the original equation. The student also has the option of letting the Solver choose the next step. Since the program works out the arithmetic details, this approach emphasizes the methods used in solving linear equations.

PROGRAM NOTES: The Solver uses the following menu.

CODE	DESCRIPTION
ADD	adds a constant or a multiple of x to both sides of the equation.
SUB	substracts a constant or a multiple of x from both sides of the equation.
MULT	multiplies both sides of the equation by a nonzero constant.
DIV	divides both sides of the equation by a nonzero constant.
ORIG	returns the equation to its original form.
CLT	combines all like terms.
DIST	applies distributive axiom; converts expressions of the form $a(b + c)$ to $ab + ac$.
NEXT	requests the computer to choose and execute the next step.

As in Using One Operation, the Solver will not execute a request to multiply or divide by zero. Also, input numbers must not exceed 32,768. If an operation produces a calculation exceeding this limit, an Out of Range message is printed, accompanied by a beep, and the original equation is displayed.

Here are some additional Solver notes:

- To ADD or SUB $-x$, enter as $-1x$. When entering a variable with a fractional coefficient, use the form $2x/3$ (not $2/3\ x$).
- If you select DIST when there are no expressions in the form $a(b + c)$ or CLT when there are no like terms to be combined, the last equation will be repeated.
- When you choose NEXT, the step being executed by the program will be described at the top of the screen.
- To see a description of each operation, ask for HELP. (? on Apple IIe, IIc; <escape>? on Apple II, II+). Only the last equation will reappear after HELP is requested; all earlier steps are erased. (Remember that you can retrieve the original equation by using the ORIG command.)

ACTIVITIES

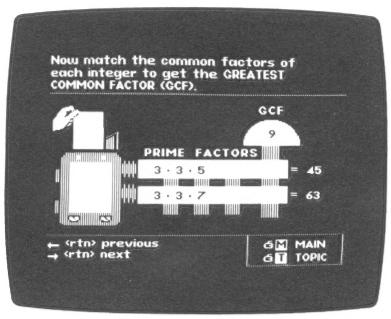
OBJECTIVE: To improve speed and accuracy in solving linear equations.

INTRODUCTION: For a description of Drive to Glory, see the discussion of ACTIVITIES on pages 4-5. Five levels of difficulty are available for both Drive to Glory and Practice Problems, as described above.

DISK 2, PART 1:
Multiplying and Factoring Polynomials

FOCUS
- Greatest Common Factor Polynomials
  - Multiplying
  - Factoring

ACTIVITIES
- The Amazing Game
- Practice Problems



GREATEST COMMON FACTOR

OBJECTIVE: To name the greatest common factor of two monomials.

INTRODUCTION: Through a "factoring machine" (see below left), the student is shown how to derive the greatest common factor of two numbers by inspecting prime factors. Similarly, the machine is used to demonstrate the GCF of two monomials.

You may want to review the meaning of prime number before using this tutorial, pointing out that 1 is not considered to be a prime number. Have students factor a number such as 120 as a product of primes; ask if there are other ways to factor 120, using only prime numbers. Students should understand that every number has a unique factorization into primes.

PROGRAM NOTES: Remember that 1 is not prime, so that the factoring machine will show no prime factors if you enter 1. To enter variable expressions with exponents, remember to use ^ (<shift>6 on Apple IIc, IIe; <shift>N on Apple II, II+). The cursor will move up to the exponent position and immediately return to the base line after the exponent is entered. For the practice section on factoring the GCF from a binomial, be sure to enter the terms in the original order. For example, for $52x + 26 = 26(?)$, the correct response is $2x + 1$, not $1 + 2x$. Also note that parentheses should not be entered as part of the answer here.

SUGGESTED EXTENSION: Have students find the GCF of (1) a polynomial with 3 or more terms, or (2) two monomials, each having more than 1 variable raised to a power.

POLYNOMIALS: MULTIPLYING

OBJECTIVE: To find the product of two binomials.

INTRODUCTION: The distributive axiom is applied to the product of two binomials. General forms are derived for the products:

$$(a + b)(c + d)$$
$$(a - b)(c - d)$$
$$(a + b)(c - d)$$
$$(a + b)(a + b)$$
$$(a - b)(a - b)$$
$$(a + b)(a - b)$$

Since this tutorial presents an application of the distributive axiom, students should understand the axiom before using this section. (Disk 1 of ALGEBRA ACTION contains a tutorial on the distributive axiom.)

POLYNOMIALS: FACTORING

OBJECTIVE: To factor a polynomial as a product of two binomials.

INTRODUCTION: In this section, the student learns to factor trinomials by first removing the GCF and then testing to see if the resulting trinomial can be expressed as the product of two binomials. For the polynomial $ax^2 + bx + c$, the program constructs a table of factors of a and c to investigate possible binomial factors.

PROGRAM NOTES: In the "warm-ups," when asked for a pair of factors, you must enter both factors, separated by a comma, before pressing <return>. Plus signs are not accepted.

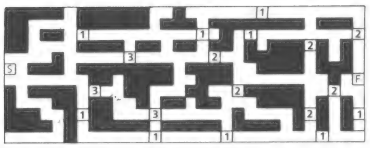
Problems of the form $21x^2 + 14x - 7 = 7(?)(x + 1)$ are presented in the last part of this tutorial. If an incorrect answer is entered, the program displays the factor table for a and c, along with the correct answer.

SUGGESTED EXTENSION: Ask students to find the missing constant term in a trinomial square, a polynomial that has 2 identical binomial factors. They should (1) factor out the GCF, (2) multiply the coefficient of the linear term by 0.5, and (3) square the result.

Here are some examples.

$x^2 + 8x + ?$ (16) $z^2 - 22z + ?$ (121) $3a^2 + 42a + ?$ (147)

Mention that students are *completing the square.*



ACTIVITIES

OBJECTIVE: To improve speed and accuracy in multiplying and factoring binomials.

INTRODUCTION: In The Amazing Game, the student works through a maze by solving problems at any of three levels of difficulty. An optional timer may be used to keep track of how fast the student solves the maze. The Practice Problems section presents the same types of questions, without the maze format.

Your location in the maze is shown by the flashing cursor, and the finish is designated by the letter F. Each number represents an obstacle—a question to be answered—with the number signifying the level of difficulty.

LEVEL	SAMPLE PROBLEM
1	Find the GCF of $5x^2$ and $6x^3$.
2	$(x - 11)(x - 11) = ?$
3	$49x^2 + 56x = 7x(?)$

When you hit an obstacle, you will hear a beep, and a problem is displayed; to clear the obstacle, you must solve the problem correctly. If you answer the problem incorrectly, you may try the same obstacle again (the program will generate a new question for you), or you may move the cursor to a new location. If you have chosen to use the timer, it will record the seconds taken both to move the cursor and to answer the question; while screens are being drawn, your time remains fixed. Note that you must tap the appropriate keys to move the cursor; holding a key down has no effect.

DISK 2, PART 2:
Fractions, Ratios, and Proportions

FOCUS
- Algebraic Fractions
  - Simplifying Fractions
  - Multiplying and Dividing Fractions
  - Adding and Subtracting Fractions
- Ratios, Proportions
- Percent Problems
- Mixture Problems
- Work Problems

ACTIVITIES
- The Amazing Game
- Practice Problems

ALGEBRAIC FRACTIONS

This FOCUS topic has a submenu with three subtopics. To exit from the submenu, press T (<escape>T on Apple II, II+).

Simplifying Fractions

OBJECTIVE: To simplify algebraic fractions by factoring and canceling.

INTRODUCTION: Students are taught to simplify fractions such as

$$\frac{x^2 - 9}{x^2 + 6x + 9}$$

by first factoring both numerator and demoninator, then canceling where possible. Multiplicative opposites are reviewed, showing the student that the quotient of a number and its opposite can be simplified as -1.

Since Algebraic Fractions extends the cancellation rule for fractions to algebraic fractions, students should understand canceling before using this tutorial.

Multiplying and Dividing Fractions

OBJECTIVE: To multiply and divide fractions with binomial expressions.

INTRODUCTION: The rule for multiplication of fractions is extended to algebraic fractions. Also, the procedures for simplifying fractions are incorporated into the multiplication process. Finally, division is presented as multiplication by a reciprocal.

You may wish to prepare students for the ideas in this section by asking them to find fractional parts of fractions, such as $\frac{2}{3}$ of $\frac{1}{2}$. Ask them to state a rule for multiplication of fractions. Similarly, introduce division of fractions, using questions such as $\frac{2}{3} \div \frac{1}{6} = \underline{?}$. Make sure that students using this tutorial understand what the reciprocal of a fraction is.

PROGRAM NOTES: Because of the nature of the questions in this section, considerable variation in input is acceptable. For example, for the question

$$\frac{2x - 4}{5x + 5} \div \frac{4x + 4}{2x - 4} = \frac{?}{?},$$

acceptable solutions include

$$\frac{(x - 2)^2}{5(x + 1)^2}, \quad \frac{(x - 2)(x - 2)}{(5x + 5)(x + 1)}, \text{ and } \quad \frac{x^2 - 4x + 4}{5x^2 + 10x + 5}.$$

However, since numerators and denominators are treated separately, *you must cancel whenever possible.* Thus,

$$\frac{4(x - 2)^2}{20(x + 1)^2}$$

would not be an acceptable answer for the problem above.

Adding and Subtracting Fractions

OBJECTIVE: To add and subtract algebraic fractions with different denominators.

INTRODUCTION: The student learns to add fractions by finding their least common denominator, expressing each fraction in terms of the LCD, and then adding and simplifying. Typical problems for this tutorial include:

$$\frac{3x + 4}{4x + 6} - \frac{5x + 1}{8x + 12} = \frac{?}{?} \qquad \frac{5x - 5}{6x + 1} + \frac{2x + 3}{x - 7} = \frac{?}{?}$$

PROGRAM NOTES: As with multiplication and division of fractions, considerable variation in responses is acceptable. Because numerators and denominators are treated separately, however, *canceling must be done whenever possible.* (See PROGRAM NOTES for Multiplying and Dividing Fractions, above, for examples of variation in input.) If either part of the response is incorrect, the program displays a 3 or 4 step solution. Note that the computer algorithm for adding and subtracting fractions may not always utilize the most efficient method; for the sake of clarity, simplification is left for the final steps. Student methods may vary from those shown.

RATIOS, PROPORTIONS

OBJECTIVE: To express ratios in simplest terms and to solve proportion problems.

INTRODUCTION: Ratios are illustrated through a series of interactive examples. Later examples require conversion of units and reducing each ratio to simplest terms.

A proportion is defined to be an equality between two ratios. In any proportion, the product of the means equals the product of the extremes; students apply this rule to test the validity of proportions. Finally, proportions are used to solve for unknown variables.

PROGRAM NOTES: Be sure to enter all ratios in simplest terms. For the O:X ratios, enter the whole ratio (for example, 7:13); if only one term is entered, the answer will be treated as an incorrect response. For the cost of tickets in the proportion problems, enter the price as a whole number, without $ or any other symbols.

PERCENT PROBLEMS

OBJECTIVE: To solve simple word problems involving percent.

INTRODUCTION: First, the student practices solving problems such as:

34% of 89 is ?
5 is 27% of ?
What percent of 48 is 16?

Next, the tutorial shows how to compute the percent of increase or decrease and applies this procedure to solving word problems involving discounts.

PROGRAM NOTES: When answering questions about % of increase or decrease, enter only a whole number. For the discount problems in which a price is requested, enter the amount without a $—for example, 10.69. (Note that decimal points *are* accepted as input in these problems.)

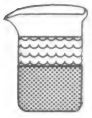
MIXTURE PROBLEMS

OBJECTIVE: To solve problems involving mixing and diluting solutions.

INTRODUCTION: This tutorial uses illustrations to demonstrate both the mixing and diluting of solutions. Students are led to formulate equations in order to solve problems such as:

> You want to produce 45 mL of a 63% solution by mixing a 39% solution and a 79% solution. How many mL of 39% solution do you need?

25 mL of
60% iodine solution



10 mL water
15 mL iodine

A beaker is used to demonstrate the effect of diluting an iodine solution and of increasing the iodine in a solution. Beginning with a uniform mixture ( ), the iodine ( ) settles to the bottom, with the water ( ) at the top. While the on-screen beaker illustrates changes in the solution, the student is guided through the corresponding mathematical changes. Students then practice solving problems such as:

> How many mL of iodine must you add to 15 mL of a 10% solution to get a 40% solution?

PROGRAM NOTES: As in most sections, you should not attempt to enter decimal points as part of your answers. All answers should be rounded to the nearest integer.

CHALLENGING THOUGHT: No matter how much iodine you add, the resulting mixture will still have some water in it (100% − 80% = 20%; 0.20 × 30 mL = 6 mL water). Thus, it is not possible to get a 100% iodine solution.

WORK PROBLEMS

OBJECTIVE: To solve work problems involving two different rates.

INTRODUCTION: This section of the program first demonstrates how to derive a work rate as amount of job per unit of time. The student then learns how to find a combined rate, and finally to solve an equation for the total time required for two people working together at different rates.

Students often find work problems confusing because the associated times are not additive. In this tutorial, they are led step by step to a valid way of finding a combined time. You can help prepare them for this investigation by giving them a simple work problem, such as:

> Viola can mow the lawn in 4 hours, and Evelyn can mow it in 6 hours. If they work together, will it take them less than 4 hours, between 4 and 6 hours, or more than 6 hours? (Answer: less than 4 hours)

Be sure the students understand why a combined time must be less than the time required by either person working alone.

PROGRAM NOTES: For all questions in this section, answers must be input in the form of a fraction reduced to lowest terms or a whole number. Mixed numerals and decimal forms are not accepted.

CHALLENGING THOUGHT: It takes 20 minutes to do one job, or 1 hour for 3 jobs, and 24 hours for 72 jobs. The rate is 72 jobs per day.

SUGGESTED EXTENSION: It may be helpful to organize information for work problems into tables. See, for example, Algebra, Structure and Method, Book 1, pages 312-316. Students who have completed this tutorial may wish to try the more complex problems that can be solved by this type of organization.

ACTIVITIES

OBJECTIVE: To improve speed and accuracy in solving problems involving fractions, ratios, and percents.

INTRODUCTION: For a description of The Amazing Game, see the ACTIVITIES discussion for Part 1 of Disk 2 (page 8). Typical problems for both the maze and the Practice Problems are shown below.

LEVEL	SAMPLE PROBLEM
1	$\frac{8}{16} \stackrel{?}{=} \frac{1}{3}$ (Ratios)
2	$\frac{6x - 1}{4x - 3} - \frac{x + 1}{4x + 4} = ?$ (Adding and Subtracting Fractions)
3	What percent of 74 is 56? (Percents)

See PROGRAM NOTES for Adding and Subtracting Fractions (page 9) for a complete discussion of level 2 inputs.

DISK 3, PART 1:
Graphing Linear Equations

FOCUS
- Coordinate Plane
- Graphing Linear Equations
- Solving Systems of Linear Equations
  - Graphing
  - Substitution
  - Multiplication with Addition or Subtraction

ACTIVITIES
- The Crossover Game
- Practice Problems

COORDINATE PLANE

OBJECTIVE: To name the coordinates of any point in the coordinate plane. To identify the quadrant in which a given point lies.

INTRODUCTION: The tutorial begins by building a coordinate plane and labeling its parts—*x*-axis, *y*-axis, origin, and unit length. Quadrants are also identified. The rules for naming points as ordered pairs and the terms abscissa and ordinate are explained. In the Quadrant Game, the student is given the coordinates of a point and asked to identify in which quadrant or on which axis the point lies. Each game consists of 6 questions, worth 3 points each.

Finally, students practice naming the coordinates of given points in order to construct a figure. After the student enters the coordinates of a set of points, the points are connected by line segments to reveal a figure. The program contains 10 different figures. If the student continues past the tenth figure, the figures will be shifted and repeated with new coordinates.

PROGRAM NOTES: In the Quadrant Game, use the ↑↓ keys (AZ on Apple II, II+) to move the checkmark to your answer; then press <return>. The program indicates the actual location by plotting the point and by placing a checkmark to the right of the correct answer. When entering coordinates, type in the form (*a*,*b*), using only integers for *a* and *b*.

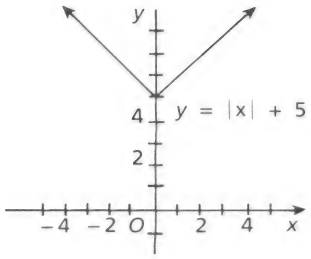
SUGGESTED EXTENSION: You may wish to have students create their own figures for plotting. Each figure should consist of a sequence of line segments, defined by the coordinates of the points which are to be connected. (Note that a set of points may define different figures when the order in which they are connected is changed.) Students may work in pairs, trying each other's figures, or you may present several as a class activity.

GRAPHING LINEAR EQUATIONS

OBJECTIVE: To graph a linear equation by plotting two of its solutions.

INTRODUCTION: The tutorial demonstrates how to find two solutions of a linear equation by replacing each variable in turn with 0 and then solving for the other variable. To graph the equation, students learn to plot the points representing the two solutions and then draw the line containing those points; a third point on the line is used as a check.

In the practice session, the student is asked to name two points on the line of a given equation. As the student answers, the points are plotted. (Only points with coordinates less than 4.7 are shown on the screen.) Then the line of the equation is graphed, so that the student may compare it to the points entered.



PROGRAM NOTES: Answers should be in the form (*a*,*b*). Use integers or fractions for *a* and *b*. The second point must be distinct from the first one. For example, if the first point entered is (3,2), then (3/1,4/2) will not be accepted for the second point.

CHALLENGING THOUGHT: No, the graph of $y = |x| + 5$ is not a straight line since not every *y* value has a unique *x* value. For example, if $y = 6$, *x* can be either 1 or -1. (See graph at left).

SOLVING SYSTEMS OF LINEAR EQUATIONS

This topic has a submenu with three subtopics. To exit from the submenu, press T (<escape>T on Apple II, II+) or M (<escape>M on Apple II, II+).

Graphing

OBJECTIVE: To find the common solution of two linear equations by inspecting their graphs.

INTRODUCTION: Graphs of two linear equations are used to illustrate the three possibilities for the plots of two straight lines in the coordinate plane.

- The lines intersect at one point; there is one solution.
- The lines are parallel; there is no solution.
- The lines coincide; there are infinitely many solutions.

The student is given two equations and asked to name the coordinates of the point of intersection of the lines. After the student answers, the point is plotted, along with the two lines. If necessary, the correct solution is shown.

PROGRAM NOTES: Students should have graph paper, ruler, and pencil to work out the solutions for the practice section. The coordinates for these solutions are integers between -5 and 5; thus, the points of intersection will always be visible on the portion of the coordinate plane shown.

Substitution

OBJECTIVE: To find the common solution of two linear equations by the substitution method.

INTRODUCTION: The substitution method of solving a system of linear equations is demonstrated with two examples. Each step in the process is displayed, along with the explanation of the method:

- solve one equation for one variable;
- substitute in the second equation;
- solve for the second variable;
- check by substitution in the original equation.

The student is asked to solve a system of two linear equations using the substitution method. If the answer is incorrect, the program will show, step by step, one way of finding the common solution.

PROGRAM NOTES: Enter answers in the form (*a*,*b*), using integers for *a* and *b*.

Multiplication with Addition or Subtraction

OBJECTIVE: To find the common solution of two linear equations by multiplication with addition or subtraction.

INTRODUCTION: Three examples are used to illustrate the methods of solving a system by multiplication with addition or subtraction. In the simplest case, one variable is eliminated when the two equations are added or subtracted; no multiplication is involved. The second and third examples show multiplication with addition and multiplication with subtraction, respectively. Each example is worked out in detailed steps. The student is then asked to solve a system of two equations. If the answer is incorrect, the program will demonstrate one way of finding the solution.

PROGRAM NOTES: Answers should be in the form (*a*,*b*). Use integers or fractions for *a* and *b*; mixed numeral forms will not be accepted. Note that if inappropriately large numbers are entered, program response may be delayed.

CHALLENGING THOUGHT: The graph does not change. If an ordered pair (x,y) is a solution of the equation $ax + by = c$, then it is also a solution of the equation $nax + nby = nc$. For example, (1,2) is a solution of

$$3x - y = 1.$$

multiply by 2: $6x - 2y = 2$

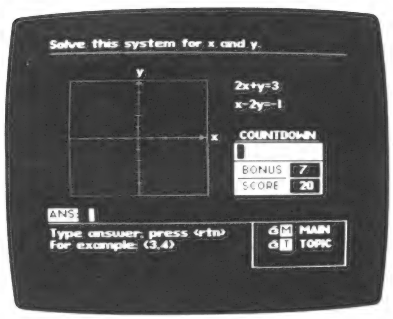
substitute (1,2): $6(1) - 2(2) \stackrel{?}{=} 2$

$2 = 2$

So (1,2) is also a solution of the new equation; both equations have the same graph.

ACTIVITIES

The Crossover Game



OBJECTIVE: To improve speed and accuracy in solving systems of linear equations.

INTRODUCTION: In this game, the student races to solve a pair of linear equations while the "countdown" bar keeps track of bonus points. Initially, the bonus is set at 10 points; each time the bar disappears, 1 point is subtracted. If the problem is correctly solved, the student earns 10 points, plus any bonus points still remaining. If an incorrect solution is entered, the program encourages the student to try one more time. Both lines are graphed and the solution of the two equations is given. After five rounds, the total score is shown, and the game may be repeated.

PROGRAM NOTES: Enter solutions in the form (a,b). Use integers for a and b.

Practice Problems

OBJECTIVE: To improve accuracy in graphing linear equations and solving systems of linear equations.

INTRODUCTION: Students are given a submenu with three types of problems. To exit from the Practice Problems submenu, press ⌘T (<escape>T on Apple II, II+) or ⌘M (<escape>M on Apple II, II+).

Graphing Linear Equations problems ask students to name two points that lie on the graph of the given equations. The two input points are plotted. (Only points with coordinates less than 4.7 are shown on the screen.) Then the line of the equation is graphed, so that the student can compare it to the points entered.

The Substitution section and the Multiplication with Addition or Subtraction section present systems of linear equations to be solved by these methods, respectively. If the student answers incorrectly, the program shows one step-by-step solution using the appropriate method.

PROGRAM NOTES: See PROGRAM NOTES (page 12) for a discussion of Graphing Linear Equations inputs. See PROGRAM NOTES (page 12) for a discussion of the Substitution inputs. See PROGRAM NOTES (page 12) for a discussion of the Multiplication with Addition or Subtraction inputs.

DISK 3, PART 2:
Introduction to Functions

FOCUS
- Slope
- Functions
- Variation

ACTIVITIES
- Graph-it
- Practice Problems

SLOPE

OBJECTIVE: To find the slope of a line. To transform a linear equation into slope-intercept form. To find an equation of a line, given two points on the line.

INTRODUCTION: A ski scene is used to introduce the concept of slope, defined as rise over run. Positive, negative, and zero slopes are illustrated. In a practice session, the student enters a rise and a run, and the program graphs the line through the origin with that slope; other lines with the same slope are then displayed. The student also practices finding slopes of lines determined by the origin and another point in the plane.

Slope is also shown as the ratio of the difference of the y-coordinates to the difference of the x-coordinates. Transforming a linear equation into slope-intercept form is demonstrated as a way to make graphing easier. The student practices transforming equations from standard form, $Ax + By = C$, to slope-intercept form, $y = mx + b$. After the slope and y-intercept are entered, the program shows the graph of the equation.

Finally, the tutorial explains how to derive an equation of a line when two points on the line are given. The student practices finding the equation determined by two points shown in the coordinate plane. The student's equation is then graphed for comparison with the original points. (If the line does not contain any points whose coordinates are between -5 and 5, then a message indicates that the graph cannot be shown on the screen.) If necessary, the correct equation and its graph are also displayed.

PROGRAM NOTES: When entering rise and run, use integers between -5 and 5, inclusive. If run is entered as 0, the program will show lines having no slope. If rise is also 0, however, the entry will be rejected. For slopes and y-intercepts, both integers and fractions may be used. Note that program response may be delayed if inappropriately large numbers are entered.

When entering the equation of a line, follow these rules:

- Write the equation in either standard form ($Ax + By = C$) or slope-intercept form ($y = mx + b$).
- Use parentheses around fractions. For example, to enter $y = \frac{1}{2}x + 2$, type $y=(1/2)x+2$.
- For coefficients, use integers between -100 and 100, inclusive, or fractions with numerators and denominators between -32 and 32, inclusive.

CHALLENGING THOUGHT: Let (a,b) and (a,c) be points on the line. The line passing through them is a vertical line with equation $x = a$.

FUNCTIONS

OBJECTIVE: To find the member of the range of a function that corresponds to a given member of the domain. To change the graph of a parabola by varying the coefficients of its quadratic equation.

INTRODUCTION: Functions are introduced as another way of viewing a straight line or a linear equation. A function is compared to a machine, with the domain being the numbers input into the machine and the range being the numbers the machine outputs. An on-screen "machine" illustrates the process; for each domain value the student inputs, the machine outputs the range value. Students also practice finding the range value of a function when the domain value is given.

In the last part of the tutorial, quadratic equations and their graphs are presented. The student enters a quadratic equation and the program graphs the corresponding parabola and shows the x-coordinate of its vertex. If the parabola does not have any points whose coordinates are between -5 and 5, a message indicates that the graph cannot be shown on the screen.

PROGRAM NOTES: When entering domain values, use integers between -20 and 20, inclusive. When entering range values, use integers between -999 and 999, inclusive. To enter a quadratic equation, follow these rules.

- Write the equation in the form $y = ax^2 + bx + c$, with $a \neq 0$.
- Use ^ to enter the exponent. Press <shift>6 on Apple IIc, IIe or <shift>N on Apple II, II+. The ^ will not appear, but the cursor will move to the exponent position. Type the exponent, 2; the cursor will then return to the base line.
- Use parentheses around fractions. For example, to enter $y = -3x^2 - \frac{3}{4}x + 1$, type $y=-3x\wedge 2-(3/4)x+1$.
- For coefficients, use integers between -100 and 100, inclusive, or fractions with numerators and denominators between -32 and 32, inclusive.

CHALLENGING THOUGHT: Substitute the range value into the function equation and solve for x. For example, let the function be $g(x) = 2x - 3$. To find the member of the domain corresponding to a range value of 9, substitute 9 for $g(x)$ and solve for x.

$$\begin{aligned} 9 &= 2x - 3 \\ 12 &= 2x \\ x &= 6 \end{aligned}$$

VARIATION

OBJECTIVE: To solve problems of direct and indirect variation.

INTRODUCTION: In this tutorial, direct variation is defined as a function given by an equation of the form $y = kx$, where k is a nonzero constant. The program demonstrates how to solve a direct variation problem by setting up a proportion and solving for the missing term. In the practice section, the student is given values such as the following and asked to find the missing term if y is directly proportional to x:

$$x_1 = ? \qquad y_1 = 15$$
$$x_2 = 45 \qquad y_2 = 25$$

A HELP screen, suggesting the equations and methods that may be used to solve these exercises, is available during the practice section. (To get HELP, press ? on Apple IIc, IIe; press <escape>? on Apple II, II+).

Similarly, an inverse variation is a function given by an equation of the form $xy = k$, where k is a nonzero constant. The program demonstrates how to solve a problem in indirect variation by using the equation $x_1x_2 = y_1y_2$. The student is then asked to solve indirect variation problems, such as:

$$x_1 = 8 \qquad y_1 = 9$$
$$x_2 = ? \qquad y_2 = 6$$

As with the direct variation problems, a HELP screen is available for these exercises.

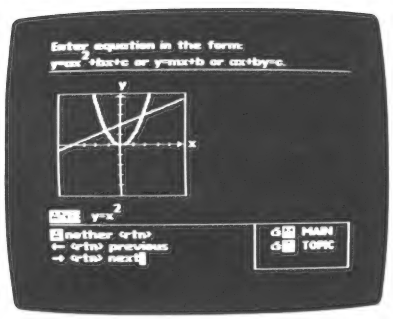
You may choose to review proportions with your students before having them use this FOCUS section. (Disk 2 contains a tutorial on ratios and proportions.)

PROGRAM NOTES: For both types of variation problems, enter integers between −999 and 999, inclusive. If an incorrect answer is entered, the program encourages you to try again.

ACTIVITIES

Graph-it

OBJECTIVE: To see the change in a graph of a linear or quadratic equation when the coefficients are varied.



INTRODUCTION: The student enters a linear or quadratic equation, and the program shows its graph. After each graph, the student chooses whether or not to clear the graph; thus, systems of two or more equations may also be graphed. The screen shows a fixed portion of the coordinate plane. If the graph of the equation entered does not contain any points within that area, a message indicates that the graph cannot be shown.

PROGRAM NOTES: Follow these rules when entering equations.

- Write a linear equation in either standard form ($Ax + By = C$) or slope-intercept form ($y = mx + b$).
- Write a quadratic equation in the form $y = ax^2 + bx + c$, with $a \neq 0$.
- Use ^ to enter the exponent. Press <shift>6 on Apple IIc, IIe or <shift>N on Apple II, II+. The ^ will not appear, but the cursor will move to the exponent position. Type the exponent, 2; the cursor will then return to the base line.
- Use parentheses around fractions. For example, to enter $y = \frac{3}{5}x - 4$, type $y=(3/5)x-4$.
- For coefficients, use integers between −100 and 100, inclusive, or fractions with numerators and denominators between −32 and 32, inclusive.

Practice Problems

OBJECTIVE: To improve accuracy in solving problems involving slope, functions, and direct variation.

INTRODUCTION: Students are given a submenu with three types of problems. To exit from the Practice Problems submenu, press either T (<escape>T on Apple II, II+) or M (<escape>M on Apple II, II+).

Slope problems consist of transforming equations into slope-intercept form. Function problems ask for the range value of a function when a domain value is given. Variation problems involve direct variation; three values are given and the fourth is to be found.

PROGRAM NOTES: For slope problems, integers and fractions may be used. Note that if inappropriately large numbers are entered, program response may be delayed. For both function and variation problems, only integers between -999 and 999, inclusive, will be accepted.

DISK 4, PART 1: Inequalities

REVIEW
- Inequality Symbols

FOCUS
- Inequalities in One Variable
- Conjunctions; Disjunctions
- Inequalities in Two Variables

ACTIVITIES
- War of Wits
- Practice Problems

INEQUALITY SYMBOLS

OBJECTIVE: To review four inequality symbols: $<$, $>$, $\leq$, $\geq$.

INTRODUCTION: Students practice filling in the blank in exercises such as $3 \ \underline{?} \ -1$ and $-2 \ \underline{?} \ -\frac{11}{4}$. Two of the four inequality symbols are displayed for each exercise, and the student chooses the one that makes a true statement.

PROGRAM NOTES: Use ↑↓ keys (AZ on Apple II, II+) to select the symbol; then press <return>. If the answer is incorrect, a checkmark will appear to the right of the correct answer.

INEQUALITIES IN ONE VARIABLE

OBJECTIVE: To solve inequalities containing one variable.

INTRODUCTION: Students are introduced to graphs of simple inequalities such as $x < 4$ and $-5 < x < -1$ and asked to state whether a given value of x satisfies each inequality. The student also practices matching an inequality to a given graph. Number line illustrations demonstrate the effect of performing the same operation on both sides of an inequality. These operations are compared to transforming equations, with one exception: the direction of the inequality symbol is reversed when both sides are multiplied or divided by a negative number. The program demonstrates how to solve inequalities by transformation. Students then use transformations to find the solution set of a given inequality. If the answer is incorrect, the program shows, step-by-step, one way of solving the inequality. The graph of the solution set is also displayed.

PROGRAM NOTES: For true/false questions, enter just one letter, T or F, and press <return>. The program accepts either a capital or lowercase letter. When selecting the inequality that matches the graph shown, use the ↑↓ keys (AZ on Apple II, II+) to move the checkmark; then press <return>.

Follow these rules when solving an inequality.
- Enter the answer in the form $x < a$. Integers or fractions may be used for a; any one of the four inequality symbols ($<$, $>$, $\leq$, $\geq$) may be used in the answer.
- To indicate $\leq$, type $<$ and then $=$; to indicate $\geq$, type $>$ and then $=$.

SUGGESTED EXTENSION: You may wish to present sentences and have students translate them into inequalities. Here are several examples.
- two less than three times a number is greater than -8 $\quad 3x - 2 > -8$
- $\frac{5}{3}$ is less than or equal to the opposite of a number $\quad \frac{5}{3} \leq -x$
- two times the sum of a number and 6 is at least two more than the number $\quad 2(x + 6) \geq x + 2$

You may also have students solve the resulting inequality, choose a value in the solution set, and check it against the original sentence.

CONJUNCTIONS; DISJUNCTIONS

OBJECTIVE: To solve conjunctions and disjunctions. To solve inequalities involving absolute value.

INTRODUCTION: Conjunctions are defined as sentences joining two open sentences by the word "and." Examples of conjunctions having zero, one, or infinitely many solutions are shown. Out of four given conjunctions, the student selects the one whose graph is shown. Later the student is asked to name a number in the solution set of a given conjunction. Disjunctions are explained similarly; the student then selects the disjunction whose graph is shown. Absolute-value inequalities are described as compact forms of conjunctions and disjunctions. The student is given four inequalities (a mixture of conjunctions, disjunctions, and absolute-value inequalities) and asked to select the one whose graph is shown.

PROGRAM NOTES: When selecting the correct conjunction, disjunction, or inequality, use the ↑↓ keys (AZ on Apple II, II+) to move the checkmark; then press <return>. When asked to name a solution of the given conjunction, enter either an integer or a fraction. If the number entered is on the portion of the number line shown, an arrow will point to that number; otherwise a message will appear. The graph of the conjunction will also be displayed.

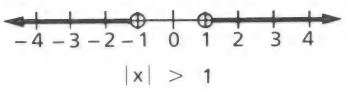
CHALLENGING THOUGHT: (1) No. The solution set of a disjunction contains the values that satisfy at least one of the inequalities. And since each one of those inequalities has an infinite number of solutions, a disjunction will always have an infinite number of solutions.

(2) No. Absolute-value inequalities with $>$, $\geq$ are equivalent to disjunctions, and absolute-value inequalities with $<$, $\leq$ are equivalent to conjunctions. Examples:

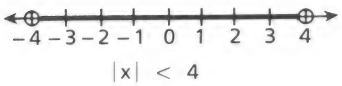
- $|x| > 3$ is equivalent to the disjunction $x > 3$ or $x < -3$, since only numbers larger than 3 or less than -3 will have absolute value greater than 3.

  $|4| = 4 > 3$; $|5| = 5 > 3$; $|-4| = 4 > 3$; $|-5| = 5 > 3$
- $|x| < 2$ is equivalent to the conjunction $-2 < x < 2$, since only numbers between -2 and 2 have absolute value less than 2.

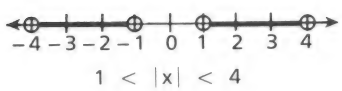
  $|-1| = 1 < 2$; $|0| = 0 < 2$; $|1| = 1 < 2$



$|x| > 1$



$|x| < 4$



$1 < |x| < 4$

SUGGESTED EXTENSION: You may wish to have students compare three graphs such as $|x| > 1$, $|x| < 4$, and $1 < |x| < 4$. Point out that the inequality $1 < |x| < 4$ is a conjunction, consisting of a disjunction ($|x| > 1$) and a conjunction ($|x| < 4$). See graphs at left.

INEQUALITIES IN TWO VARIABLES

OBJECTIVE: To graph an inequality in two variables. To solve a system of two inequalities.

INTRODUCTION: Three sets of points in a coordinate plane—the set of points on a line, the set of points above the line, and the set of points below the line—are used to derive the graph of an inequality in two variables. The student practices selecting the inequality that matches a given graph and also writing an inequality whose graph is shown. If the inequality is incorrect, the program asks the student to try again.

Systems of linear inequalities are introduced. The solution of a system is described as the set of points whose coordinates satisfy both inequalities; this set is represented by the doubly shaded region on the graph. The student enters two inequalities, and the program graphs them on the same set of axes. (Only those points with coordinates between -6 and 6 are shown.) The student then chooses which of four points is in the solution set of a given system of two inequalities.

PROGRAM NOTES: When selecting the inequality or the point in the solution set of a system, use the ↑↓ keys (AZ on Apple II, II+) to move the checkmark; then press <return>.

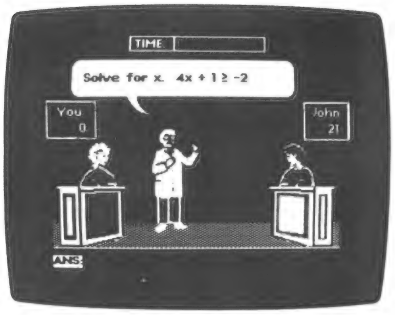
When writing an inequality or entering a system of inequalities, follow these rules.

- Write the inequality in a form similar to slope-intercept form. For example, enter as $y > -2x - 3$ (not $2x + y > -3$). For an inequality with no y term, enter in simplest form, such as $x < 5$.
- To indicate $\leq$, type $<$ and then $=$; to indicate $\geq$, type $>$ and then $=$.
- Enclose fractions for x-coefficients in parentheses. Constants should be typed without parentheses. For example, to enter $y \leq \frac{1}{2}x + \frac{5}{2}$, type y<=(1/2)x+5/2.
- Numbers larger than 500 will not be accepted.

CHALLENGING THOUGHT: A system of inequalities can contain any number of inequalities; there is no maximum.

ACTIVITIES

War of Wits



OBJECTIVE: To improve speed and accuracy in solving inequalities in one variable.

INTRODUCTION: The War of Wits is a game in which the student competes against John (the opponent simulated by the computer) in solving inequalities. The professor asks a question. To answer, the student "buzzes in" by pressing any key. The student is then allowed to enter an answer. If the student does not press a key fast enough, John will do so; then he has the opportunity to solve the inequality. Note that John does not always give the correct solution; if he doesn't, the student will have a chance to answer. If neither player solves the inequality correctly, the professor gives the answer.

A correct answer is worth 10 points. There is also a bonus of up to 20 points, for a total of 30 possible points for each correct solution. The time bar shows how much time is left for earning bonus points. The professor announces the winner after each game, which consists of 6 questions. Sound effects, which can be turned on or off, are used to enhance the game.

PROGRAM NOTES: When asked if instructions or sound is desired, enter one letter, either Y or N, and then press <return>. See PROGRAM NOTES below in the Practice Problems section (Inequalities in One Variable exercises) for a description of acceptable inputs.

Practice Problems

OBJECTIVE: To improve accuracy in solving inequalities in one variable, conjunctions, disjunctions, and inequalities in two variables.

INTRODUCTION: Students are given a submenu with three types of problems. To exit from the Practice Problems submenu, press ⌘T (<escape>T on Apple II, II+) or ⌘M (<escape>M on Apple II, II+).

In the Inequalities in One Variable section the student finds the solution set of a given inequality. If the answer is incorrect, the program shows step-by-step one way of solving the inequality. The graph of the solution set is also displayed. In the Conjunctions; Disjunctions section and in the Inequalities in Two Variables section, the student is asked to select the inequality whose graph is shown.

PROGRAM NOTES: For the Inequalities in One Variable exercises, follow these rules for writing the solution of an inequality.

- Enter the answer in the form $x<a$, where a is an integer or a fraction. Use any of the four inequality symbols ($<$, $>$, $\leq$, $\geq$) in the answer.
- To indicate $\leq$, type $<$ and then $=$; to indicate $\geq$, type $>$ and then $=$.

When selecting the correct inequality in both the Conjunctions; Disjunctions exercises and the Inequalities in Two Variables exercises, use the ↑↓ keys (AZ on Apple II, II+) to move the checkmark and then press <return>. If the answer is incorrect, a checkmark will appear to the right of the correct answer.

DISK 4, PART 2:
Irrationals

REVIEW
- Squaring Numbers

FOCUS
- Square Roots
- Pythagorean Theorem
- Working with Radicals
- Quadratic Formula

ACTIVITIES
- War of Wits
- Practice Problems

SQUARING NUMBERS

OBJECTIVE: To review the square of positive and negative numbers.

INTRODUCTION: This section reviews the meaning of squaring integers and fractions. Sample problems are shown below.

$5^2 = ?$ $\qquad (-14)^2 = ?$

$-(13)^2 = ?$ $\qquad -(-9)^2 = ?$

PROGRAM NOTES: Enter the square as an integer, with a negative sign if needed; plus signs are not accepted.

SQUARE ROOTS

OBJECTIVE: To simplify square root expressions.

INTRODUCTION: This tutorial introduces the radical sign, defining $\sqrt{x}$ as the positive number whose square is x. The student first practices finding the square root of a perfect square, such as $\sqrt{400}$ and $-\sqrt{81}$. Then the student simplifies the square root of a fraction whose numerator and denominator are perfect squares. Here are two sample exercises.

$$\sqrt{\frac{36}{289}} = ? \qquad -\sqrt{\frac{121}{25}} = ?$$

The last half of this tutorial deals with simplifying $\sqrt{x}$ when x is not a perfect square. The student learns how to locate $\sqrt{x}$ between two consecutive whole numbers. More exact values, obtained by using a calculator and a table of square roots, are illustrated. Simplifying radical expressions by factoring is explained through examples such as

$$\sqrt{54} = \sqrt{9 \cdot 6} = \sqrt{9} \cdot \sqrt{6} = 3\sqrt{6}.$$

The student practices simplifying expressions of the form $\sqrt{ab}$, where a and b are nonnegative.

PROGRAM NOTES: Throughout the tutorial enter a negative sign if needed; plus signs are not accepted. Give the square root of a fraction in simplest form. For $\sqrt{\frac{324}{64}}$, for example, enter 9/4, not 18/8. When responding to questions in the form

$\sqrt{53}$ is between which two consecutive whole numbers?,

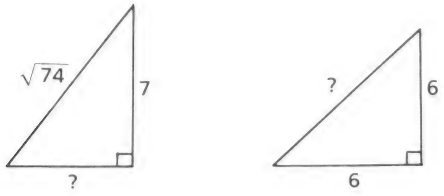
enter two integers, separated by a comma, with the smaller number first. For the question above, enter 7,8.

The answers in the last practice session require a radical sign. To enter $3\sqrt{5}$, for example, type 3, press capital or lowercase R (a radical sign will appear on the screen), and then type the radicand 5, followed by <return>. Use the simplest form of the radicand.

PYTHAGOREAN THEOREM

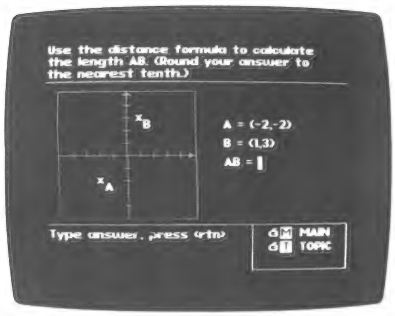
OBJECTIVE: To use the Pythagorean theorem to solve problems involving right triangles. To find the distance between two points in the coordinate plane.

INTRODUCTION: The Pythagorean theorem is stated and then illustrated by the 3,4,5 right triangle. The program demonstrates how to use the theorem to find a missing length in a right triangle. The student practices applying this method in problems such as these:



In the second half of this tutorial, the distance formula is derived from the Pythagorean theorem. The student practices finding the distance between two points whose coordinates are given.

Before using this tutorial, students should understand the use of coordinates to name points in the plane. Some students may benefit from the tutorial on the coordinate plane contained in the first part of Disk 3. You may wish to have your students use calculators or square root tables for the final step of the distance formula.


Use the distance formula to calculate the length AB. (Round your answer to the nearest tenth.)
A = (-2,-2)
B = (1,3)
AB =
Type answer, press <rtn>
MAIN
TOPIC

PROGRAM NOTES: Whenever possible, use whole numbers for the missing lengths of right triangles, as in the exercise at the left on page 19. When a radical form is necessary, write the expression in simplest form. For the exercise at the right on page 19, enter $6\sqrt{2}$ as follows: type 6, then press R and a radical sign will appear; type 2, followed by <return>.

When you calculate the length *AB* by using the distance formula, round the answer to the nearest tenth. Enter the result with a decimal point, such as 8.5, for example.

SUGGESTED EXTENSION: The program gives the 3,4,5 right triangle as an illustration of the Pythagorean theorem. Challenge your class to find other right triangles with integral sides. A set of three such integers is called a Pythagorean triple. There are 8 Pythagorean triples that have no member greater than 25.

{3,4,5}
{5,12,13}
{6,8,10}
{7,24,25}
{8,15,17}
{9,12,15}
{12,16,20}
{15,20,25}

CHALLENGING THOUGHT: (1) Yes. Suppose leg a is as long as or longer than the hypotenuse c. Then $c \leq a$ and $c^2 \leq a^2$. By the Pythagorean theorem, $a^2 + b^2 = c^2$, so substituting gives $a^2 + b^2 \leq a^2$. Then $b^2 \leq 0$, which is impossible.

(2) Since the square of any real number is nonnegative, then the sum of two squares is also nonnegative. Another way of looking at the problem: $(x_2 - x_1)^2 + (y_2 - y_1)^2$ is the expression under the radical sign in the distance formula; since distance cannot be the square root of a negative number, the expression must be nonnegative.

WORKING WITH RADICALS

OBJECTIVE: To simplify products and sums of radicals. To rationalize the denominator in a radical expression. To multiply binomials containing square-root radicals.

INTRODUCTION: This tutorial describes several types of problems containing radical expressions. Multiplying radicals with whole number radicands is demonstrated. The student simplifies products such as

$$2\sqrt{7} \cdot -7\sqrt{8}, \quad -12\sqrt{2} \cdot 9\sqrt{10}, \quad \text{and} \quad -14\sqrt{15} \cdot -4\sqrt{12}.$$

Multiplication is then extended to radicals containing fractions with perfect square denominators. Sample problems are shown below. (Some of the exercises involve only simplifying fractions.)

$$5\sqrt{\frac{7}{36}} = ? \qquad 3\frac{\sqrt{5}}{\sqrt{4}} \cdot \sqrt{\frac{9}{16}} = ? \qquad 4\sqrt{\frac{13}{25}} \cdot 2\frac{\sqrt{3}}{\sqrt{16}} = ?$$

Fractions with denominators that are not perfect squares are next included in the discussion of multiplying radicals. Several examples illustrate the process of rationalizing a denominator. The student then practices rationalizing denominators by simplifying and multiplying radical expressions. Here are several sample exercises.

$$4\frac{\sqrt{11}}{\sqrt{5}} = ? \qquad 5\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{9}{2}} = ? \qquad 2\frac{\sqrt{16}}{\sqrt{7}} \cdot 4\sqrt{\frac{15}{6}} = ?$$

Addition of radicals is then demonstrated; students learn that radicals must have the same radicand before they can be added or subtracted. The student practices finding sums such as

$$2\sqrt{12} + 3\sqrt{3}, 3\sqrt{50} + \sqrt{50}, \text{ and } 2\sqrt{112} + 5\sqrt{63}.$$

Multiplication of binomials containing square roots is the last type of problem presented. As with other binomials, the distributive axiom is used to multiply binomials involving radicals. The student is asked to fill in the missing term in a given equation; sample problems are below.

$$(\sqrt{7} - 1)(\sqrt{7} + 3) = 4 + ?$$

$$(\sqrt{13} - 3)(\sqrt{13} - 5) = ? - 8\sqrt{13}$$

PROGRAM NOTES: For all the exercises in this tutorial, follow these rules.

- Write the answer in simplest form.
- Answers may be integers, fractions, or radical expressions.
- To enter a radical, press R and a radical sign will appear on the screen. Here are two examples.

To enter:	Type in:	Appears on answer line as:
$5\sqrt{3}$	5R3	$5\sqrt{3}$
$-\frac{3}{2}\sqrt{7}$	−3/2R7	$-3/2\sqrt{7}$

QUADRATIC FORMULA

OBJECTIVE: To find the roots of a quadratic equation by using the quadratic formula.

INTRODUCTION: This final tutorial leads the student through the application of the quadratic formula.

If $ax^2 + bx + c = 0$ (with $a \neq 0$ and $b^2 - 4ac \geq 0$),

$$\text{then} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

After the program demonstrates how to use the formula to find the roots of a quadratic equation, the student practices the method by completing the process, step by step. First, a quadratic equation is presented, and the student is asked to name the coefficients a, b, and c. These values, corrected if necessary, are displayed. The student next uses these numbers to determine the value of $b^2 - 4ac$.

$$2x^2 - 4x - 13 = 0$$

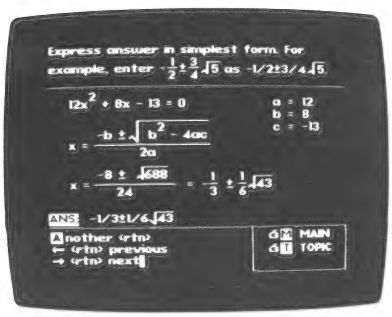
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{?}}{4}$$

$a = 2$
$b = -4$
$c = -13$

Finally, the student gives the roots in simplest form:

$$x = \frac{4 \pm \sqrt{120}}{4} = 1 \pm \frac{1}{2}\sqrt{30}$$


Express answer in simplest form. For example, enter -1/2±3/4√5 as -1/2±3/4√5.
12x² + 8x - 13 = 0
a = 12
b = 8
c = -13
x = (-b ± √(b² - 4ac))/2a
x = (-8 ± √688)/24 = 1/3 ± 1/6√43
ANS: -1/3±1/6√43
Another <rtn>
← <rtn> previous
→ <rtn> next
MAIN
TOPIC

PROGRAM NOTES: When entering the coefficients a, b, and c, use a negative sign if necessary; plus signs are not accepted. Enter the roots in the form

$$k \pm m \qquad \text{or} \qquad k \pm m\sqrt{n}$$

where k and m are integers or fractions, and n is a whole number. Fractions and radicals must be in simplest form. To enter ±, press S. To enter the radical sign, press R, then enter the radicand. Note that the radical sign will be automatically extended as long as necessary. The table on the next page shows the formats of some typical answers. Use either capital or lowercase letters for R and S.

Roots	Type in:	Appears on answer line as:
$\frac{2}{3} \pm \frac{1}{3}\sqrt{43}$	2/3S1/3R43	$2/3 \pm 1/3\sqrt{43}$
$-7 \pm 3\sqrt{6}$	−7S3R6	$-7 \pm 3\sqrt{6}$
$\frac{3}{4} \pm \frac{5}{4}$	3/4S5/4	$3/4 \pm 5/4$

ACTIVITIES

War of Wits

OBJECTIVE: To improve speed and accuracy in solving problems involving square roots, distance between two points, binomials containing radicals, and the quadratic formula.

INTRODUCTION: See INTRODUCTION (page 18) for a description of the War of Wits game. The professor asks 6 different types of questions.

	SAMPLE PROBLEMS
1	$\sqrt{25} = ?$
2	$\sqrt{\frac{49}{100}} = ?$
3	$\sqrt{63} = ?$
4	Find the distance between (−2,3) and (1,4).
5	$(\sqrt{7} + 2)(\sqrt{7} + 4) = 15 + ?$
6	Solve for x: $x^2 - 2x - 5 = 0$

PROGRAM NOTES: When asked if instructions or sound is desired, enter one letter, either Y or N, and then press <return>. For problems 1 and 2 above, enter the answer as a whole number or a fraction in simplest form. For questions 3 and 5, a radical sign may be required in the answer. To enter $5\sqrt{7}$, for example, type 5, press R and a radical sign will appear, and then type 7, followed by <return>. Use the simplest form of the radicand.

The answer to question 4 should be rounded to the nearest tenth and entered as a decimal. See PROGRAM NOTES (pages 21-22) under Quadratic Formula for a description of acceptable inputs for problem 6.

Practice Problems

OBJECTIVE: To improve accuracy in solving problems involving square roots, distance between two points, multiplication of radicals, and the quadratic formula.

INTRODUCTION: Students are given a submenu with four types of problems. To exit from the Practice Problems submenu, press Apple T (<escape>T on Apple II, II+) or Apple M (<escape>M on Apple II, II+).

In the Square Roots section the student simplifies a radical expression of the form $\sqrt{ab}$, where a and b are nonnegative. In the Pythagorean Theorem section the student practices finding the distance between two points whose coordinates are given. Multiplying radical expressions is the focus of the exercises in the Working with Radicals section. The student finds the roots of a quadratic equation in the Quadratic Formula section.

PROGRAM NOTES: The answers to the Square Roots and the Working with Radicals exercises may require a radical sign. To enter $4\sqrt{3}$, for example, type 4, press R and a radical sign will appear, and then type 3, followed by <return>. Use the simplest form of the radicand.

When determining the distance in the Pythagorean Theorem exercises, round the answer to the nearest tenth. Enter the answer as a decimal. See PROGRAM NOTES (pages 21-22) for a discussion of the acceptable inputs for the Quadratic Formula exercises.

V. CORRELATION TO ALGEBRA 1 TEXTBOOKS

Topic on disk covered in lesson:	*Algebra, S & M, Book 1* ©1988, 1986	*Algebra 1* ©1986	*Basic Algebra* ©1988	*Elementary Algebra,*	
				Part 1 ©1988, 1985	*Part 2* ©1988, 1985
Disk 1, Part 1					
What are Variables?	1-1, 2-7	1-4, 1-5, 4-6	1-1, 1-2	1-3, 3-3	use with any lesson
Distributive Axiom and Combining Like Terms	2-5, 2-6	2-3, 2-8, 2-9	1-3, 1-7, 3-5, 3-7	6-4, 6-5, 7-1, 10-4	2-2
Word Problem Warm-ups	1-4	1-8	2-7	1-5, 5-5	use with any lesson
Disk 1, Part 2					
Signed Number Practice	2-3, 2-4, 2-6, 2-8, 2-9	2-6, 2-8, 2-9, 2-10, 2-11	3-3, 3-4, 3-6, 3-8	9-1, 9-4, 9-5, 10-1, 10-3, 10-5, 10-6	1-6, 1-7, 1-8, 1-9
Using One Operation	3-1, 3-2	3-1, 3-2, 3-3	2-1, 2-2, 2-3, 3-9	1-6, 1-7, 5-3, 5-4, 7-2, 7-3, 7-4, 11-1, 11-2	2-4, 2-5
Using Several Operations	3-3, 3-5	3-4, 3-6	2-4, 2-6, 2-9, 2-10, 3-9	7-3, 7-4, 7-5, 7-7, 11-3, 12-9	2-6, 2-7
Disk 2, Part 1					
Greatest Common Factor	5-1, 5-2, 5-3	7-5, 7-6	6-1, 6-2	14-1, 14-2, 14-3	4-4, 4-5, 4-6
Multiplying Polynomials	4-6, 5-4, 5-5, 5-6	7-3, 7-4	5-6, 5-7, 5-8, 6-8	13-5, 13-6	3-6, 3-7, 5-1, 5-2, 5-3
Factoring Polynomials	5-7, 5-8, 5-9, 5-11	7-8, 7-9	6-4, 6-5, 6-6, 6-7, 6-10, 6-11	14-8, 14-9	5-8, 5-9, 5-10, 5-11

Topic on disk covered in lesson:	*Algebra, S & M, Book 1* ©1988, 1986	*Algebra 1* ©1986	*Basic Algebra* ©1988	*Elementary Algebra,*	
				Part 1 ©1988, 1985	*Part 2* ©1988, 1985
Disk 2, Part 2					
Simplifying Fractions	6-1	8-4	9-2, 9-3		6-1, 6-3
Multiplying & Dividing Fractions	6-2, 6-3	8-6	9-8, 9-9		7-1, 7-2, 7-3, 7-4, 7-5
Adding & Subtracting Fractions	6-4, 6-5	8-7, 8-8	9-10, 9-11, 9-12, 9-13, 9-16		8-1, 8-2, 8-3, 8-4, 8-5
Ratios, Proportions	7-1, 7-2	9-7	9-4, 9-5, 9-6, 9-7	3-5	6-4, 6-5
Percent Problems	7-5, 7-6	9-2	10-6, 10-7, 10-8	4-4	6-7
Mixture Problems	7-7	4-9	10-11		Challenge, p. 185
Work Problems	7-8	9-5	9-15		8-10
Disk 3, Part 1					
Coordinate Plane	8-2	5-1	7-3	2-6	4-7
Graphing Linear Equations	8-2	5-5	7-5		9-2, 9-3
Solving Systems					
Graphing	8-3	6-1	8-1, 8-2		11-1, 11-2
Substitution	8-4	6-4	8-3		11-6
Multiplication with Addition or Subtraction	8-6, 8-7	6-2, 6-3	8-4, 8-6		11-4, 11-5

Topic on disk covered in lesson:	*Algebra, S & M, Book 1* ©1988, 1986	*Algebra 1* ©1986	*Basic Algebra* ©1988	*Elementary Algebra,*	
				Part 1 ©1988, 1985	*Part 2* ©1988, 1985
Disk 3, Part 2					
Slope	9-1, 9-2, 9-3	5-7, 5-8, 5-9	7-6	Challenge, p. 342	9-4, 9-5, 9-6, 9-7
Functions	9-4, 9-6	5-2, 5-3, 11-4	7-7, 12-8	4-8, 6-7, 9-6, 10-7	2-8, 4-8, 10-6, Challenge, p. 305
Variation	9-7, 9-8	9-8, 9-9			Challenge, p. 245 Challenge, p. 275
Disk 4, Part 1					
Inequality Symbols	1-9	1-1	1-11, 3-1	2-4, 8-3	1-2
Inequalities in One Variable	10-1, 10-2	4-2	3-2	5-7, 5-8, 8-5, 11-5, 11-6	10-1, 10-2
Conjunctions; Disjunctions	10-4, 10-5, 10-6	4-4, 4-5			
Inequalities in Two Variables	10-7, 10-8	5-6, 6-8	7-8		10-3, 10-4, 10-5, 11-7, 11-8
Disk 4, Part 2					
Squaring Numbers	4-1	1-6	1-5, 3-6	1-8	3-4
Square Roots	11-3, 11-4	10-3, 10-4	11-1, 11-2, 11-4, 11-7		12-3, 12-4, 12-5
Pythagorean Theorem	11-6, Extra, p. 525	10-5, 10-6	11-6		12-7
Working with Radicals	11-7, 11-8, 11-9	10-7, 10-8	11-8, 11-9, 11-10, 11-11		13-1, 13-2, 13-3
Quadratic Formula	12-3	11-2	12-6		13-8